Discrete Fourier Transform

Lesson 11

5DT

Homework

- Problems 6.2-4

6.2 Find the DFT of the sequence, $x[n]=\sin(\frac{n\pi}{N})$ for n = 0,1,2,...N-1.

Hint express the sinusoid in exponential form.

6.3a)Find the DFT of the sequence $x[n]=e^{-bnT}$ in closed form. Use the identity $\sum_{n=0}^{N-1}\rho^{n}\equiv\frac{1-\rho^{N}}{1-\rho}\quad 0<\rho<1;$

Note that $N\rightarrow\infty;\sum_{n=0}^{N-1}\rho^{n}\rightarrow\frac{1}{1-\rho}$

b) Express the DFT in polar form as a function of k.

c) Plot the DFT in terms of its magnitude and its phase angle for T = .2, b = 1 and N = 8 for k = 0,1,2,..7

6.4a) Repeat the 6.3 for b = 10; T = .01, N = 20

6.6 Find the DFT for the following signals both are sampled at $1m\sec$ intervals:

a) $x(nT_s)=x[n]=1; 0\leq n\leq 4$

Number of samples = 5

b) $x(nT_s)=x[n]=1; 0\leq n\leq 4$

$=0;\ 9>n>4$

Number of samples = 10

Homework

- Using Matlab and its FFT function calculate and plot the time signal and spectrum for the following single.
  - A single sine wave of frequency 200 Hz
  - A single square wave of frequency 200 Hz
  - Two simultaneous sine wave of frequency 200 and 200/3 Hz
  - Two sequential sine wave of frequency 200 and 200/3 Hz
  - Compare the spectrum of the latter two cases.

Homework Answers

- Problems 6.2

$$\mathbf{X}[k] = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(t) = \sin(\omega t)$$

$$x(nT_s) = \sin(\omega n T_s) = \sin(\hat{\omega} n)$$

$$\hat{\omega} = \omega T_s = 2\pi f_{\max} T_s = \frac{2\pi f_{\max}}{f_s} = \frac{2\pi f_{\max}}{\geq 2 f_{\max}} \leq \pi$$

$$\hat{f} = \frac{\hat{\omega}}{2\pi} \leq \frac{\pi}{2\pi} \leq \frac{1}{2}$$

$$x[n] = \sin(\frac{\pi}{N} n) = \frac{e^{j\pi n/N} - e^{-j\pi n/N}}{2j}$$

$$\hat{f} = \frac{\hat{\omega}}{2\pi} = \frac{1}{2N}$$

$$\mathbf{X}[k] = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N} = \sum_{n=o}^{N-1} \frac{e^{j\pi n/N} - e^{-j\pi n/N}}{2j} e^{-jk2\pi n/N}$$

$$= \sum_{n=o}^{N-1} \frac{e^{-[j(2k-1)\pi/N]n} - e^{-[j(2k+1)\pi n/N]n}}{2j}$$

NOTE :

$$\sum_{0}^{N-1} a^n = 1 + a + a^2 + \cdots a^{N-1}$$

$$\frac{1}{1-a} = 1 + a + a^2 + \cdots a^{N-1} + \frac{a^N}{1-a}$$

$$\frac{1-a^N}{1-a} = 1 + a + a^2 + \cdots a^{N-1}$$

$$\sum_{0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\mathbf{X}[k] = \frac{1}{2j}[\frac{1-e^{-[j(2k-1)\pi/N]N}}{1-e^{-[j(2k-1)\pi/N]}} - \frac{1-e^{-[j(2k+1)\pi/N]N}}{1-e^{-[j(2k+1)\pi/N]}}];\ \text{using} \sum_{0}^{\text{N-1}} \rho^{\text{n}} = \frac{1-\rho^{\text{N}}}{1-\rho}$$

$$= \frac{1}{2j}[\frac{1-e^{-j(2k-1)\pi}}{1-e^{-[j(2k-1)\pi/N]}} - \frac{1-e^{-j(2k+1)\pi}}{1-e^{-[j(2k+1)\pi/N]}}]$$

Homework Answers

- Problems 6.2

$$\mathbf{X}[k]=\sum_{n=0}^{N-1}x(n)e^{-jk2\pi n/N}$$

$$\mathbf{X}[k]=\frac{1}{2j}[\frac{1-e^{-j(2k-1)\pi}}{1-e^{-[j(2k-1)\pi/N]}}-\frac{1-e^{-j(2k+1)\pi}}{1-e^{-[j(2k+1)\pi/N]}}]$$

$(2k-1)\pi=-1\pi,\pi,3\pi,...,(2N-3)\pi$ for $k=0,1,2,3,...N-1$

$(2k+1)\pi=1\pi,3\pi,...(2N+3)\pi$ for $k=0,1,2,3,...N-1$

i.e., odd multiples of $\pi = p\pi$ where p is odd $\Rightarrow, 1-e^{-jp\pi}=1-(-1)=2$

$$\mathbf{X}[k]=\frac{1}{2j}[\frac{2}{1-e^{-[j(2k-1)\pi/N]}}-\frac{2}{1-e^{-[j(2k+1)\pi/N]}}]$$

$$=-\frac{1}{2}[\frac{1}{e^{-j\frac{(2k-1)}{2N}\pi}(\frac{e^{+j\frac{(2k-1)}{2N}\pi}-e^{-j\frac{(2k-1)}{2N}\pi}}{2j})}-\frac{1}{e^{-j\frac{(2k+1)}{2N}\pi}(\frac{e^{+j\frac{(2k+1)}{2N}\pi}-e^{-j\frac{(2k+1)}{2N}\pi}}{2j})}]$$

$$=-\frac{1}{2}[\frac{1}{e^{-j\frac{(2k-1)}{2N}\pi}\sin(\frac{(2k-1)}{2N}\pi)}-\frac{1}{e^{-j\frac{(2k+1)}{2N}\pi}\sin(\frac{(2k+1)}{2N}\pi)}]$$

Homework Answers

- Problems 6.2

$$\mathbf{X}[k] = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$\mathbf{X}[k] = \frac{1}{2}[\frac{e^{j\frac{(2k+1)}{2N}\pi}}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{e^{j\frac{(2k-1)}{2N}\pi}}{\sin(\frac{(2k-1)}{2N}\pi)}]$$

$$= \frac{1}{2}[\frac{\cos(\frac{(2k+1)}{2N}\pi) + j\sin(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi) + j\sin(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)}]$$

$$= \frac{1}{2}[\frac{\cos(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} + \frac{j\sin(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)} - \frac{j\sin(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)}]$$

$$= \frac{1}{2}[\frac{\cos(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)} + j\frac{\sin(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - j\frac{\sin(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)}]$$

$$= \frac{1}{2}[\frac{\cos(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)} + j - j]$$

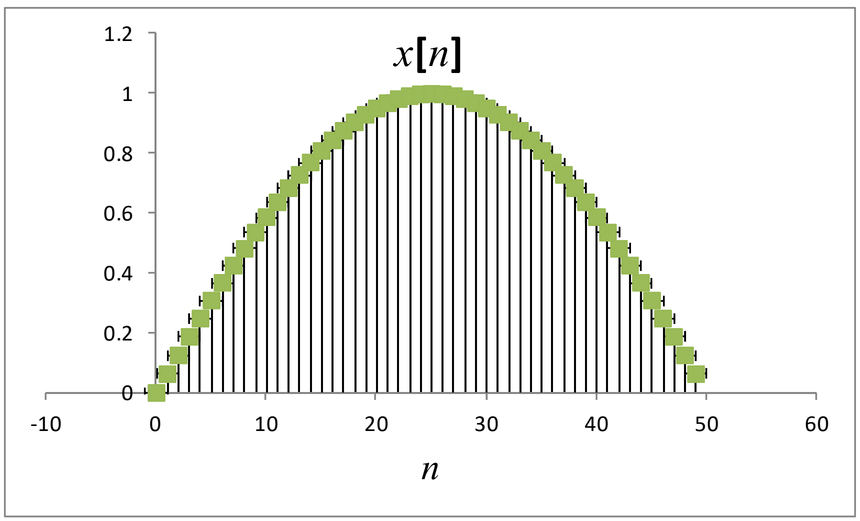
$$= \frac{1}{2}[\frac{\cos(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)}]$$

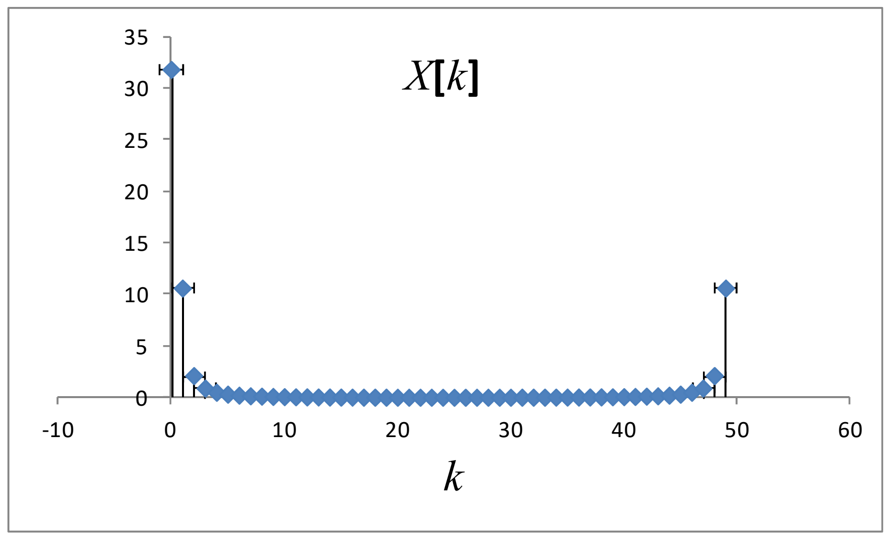
Homework Answers

- Problems 6.2

$$\mathbf{X}[k] = \sum_{n=0}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$\mathbf{X}[k] = \frac{1}{2}\left[\frac{\cos(\frac{(2k+1)}{2N}\pi)}{\sin(\frac{(2k+1)}{2N}\pi)} - \frac{\cos(\frac{(2k-1)}{2N}\pi)}{\sin(\frac{(2k-1)}{2N}\pi)}\right]$$





Homework Answers

- Problems 6.3a

$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(n) = \sum_{n=o}^{N-1} \mathbf{X(k)} e^{jk2\pi n/N}$$

$$x(n) = e^{-bnT}$$

$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$= \sum_{n=o}^{N-1} e^{-bnT} e^{-jk2\pi n/N}$$

$$= \sum_{n=o}^{N-1} e^{-(jk2\pi/N+bT)n}$$

$NOTE:$

$$\sum_{0}^{N-1} a^n = 1 + a + a^2 + \cdots a^{N-1}$$

$$\frac{1}{1-a} = 1 + a + a^2 + \cdots a^{N-1} + \frac{a^N}{1-a}$$

$$\frac{1-a^N}{1-a} = 1 + a + a^2 + \cdots a^{N-1}$$

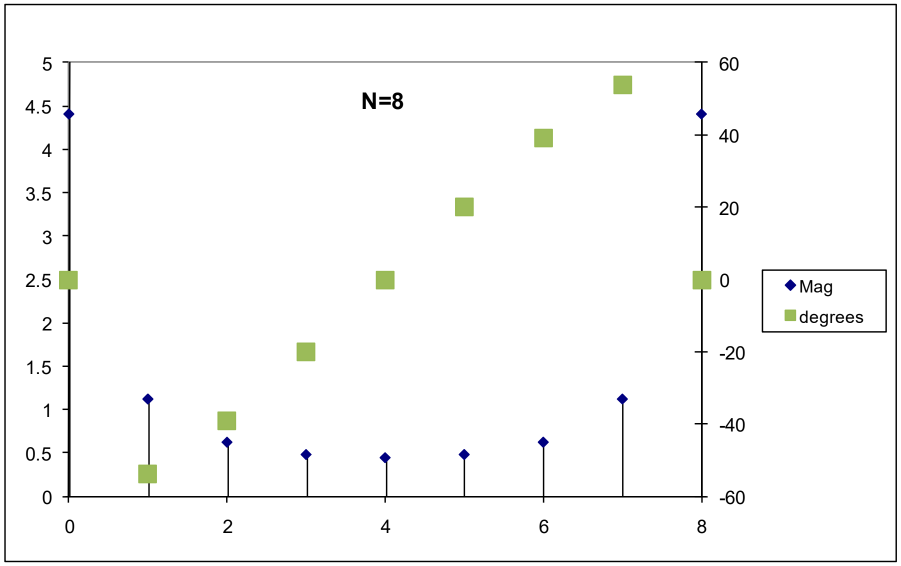
$$\sum_{0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\mathbf{X(k)} = \frac{1-e^{-(jk2\pi/N+bT)N}}{1-e^{-(jk2\pi/N+bT)}}; \text{using} \sum_{0}^{\text{N-1}} \rho^{\text{n}} = \frac{1-\rho^{\text{N}}}{1-\rho}$$

$$= \frac{1-e^{-(jk2\pi)}e^{-bTN}}{1-e^{-(jk\frac{2\pi}{N})}e^{-bT}} = \frac{1-e^{-bTN}}{1-e^{-(jk\frac{2\pi}{N})}e^{-bT}}; \text{using } e^{-(jk2\pi)} = \cos k2\pi - j\sin k2\pi = 1$$

Homework Answers

- Problems 6.3b



$$\mathbf{X(k)} = \frac{1-e^{-bTN}}{1-e^{-(jk\frac{2\pi}{N})}e^{-bT}}$$

$$= \frac{1-e^{-bTN}}{1-e^{-bT}(\cos\frac{2\pi k}{N} - j\sin\frac{2\pi k}{N})}$$

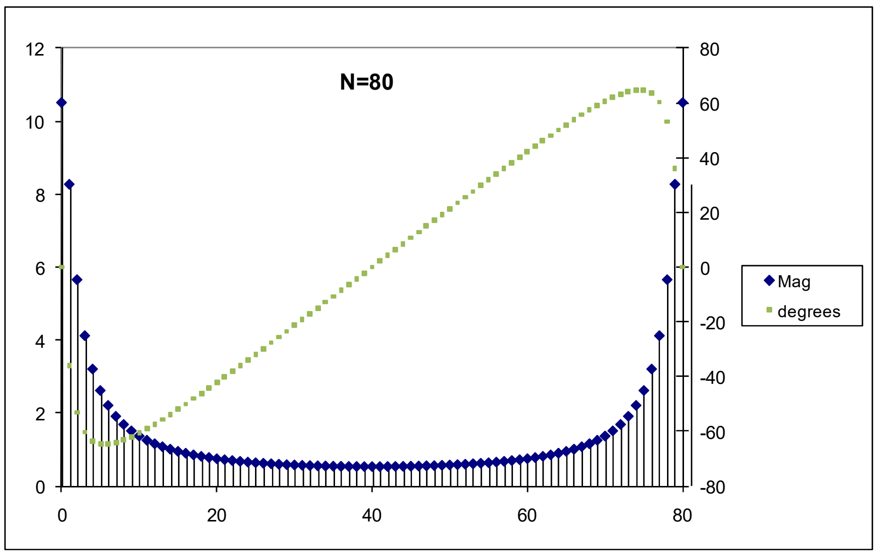
$$= \frac{1-e^{-bTN}}{1-e^{-bT}\cos\frac{2\pi k}{N} + je^{-bT}\sin\frac{2\pi k}{N}}$$

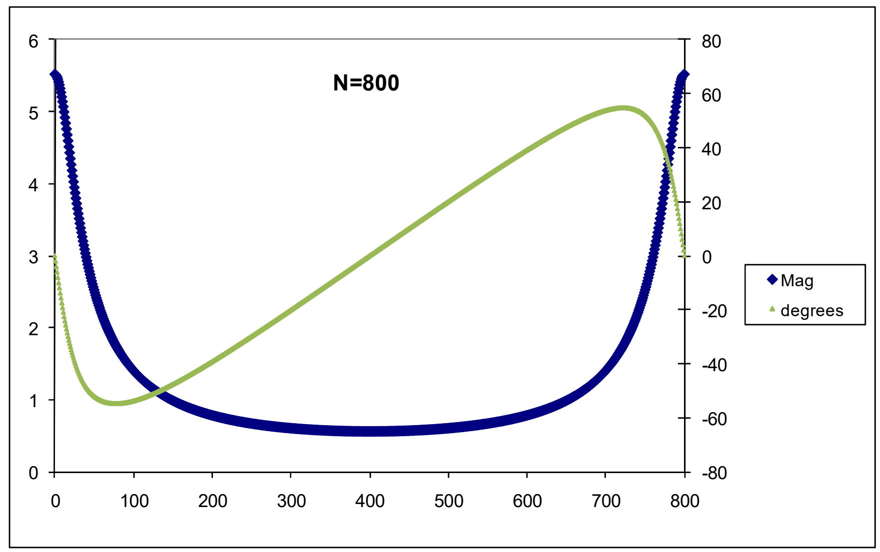
$$= \frac{1-e^{-bTN}}{\sqrt{(1-e^{-bT}\cos\frac{2\pi k}{N})^2 + (e^{-bT}\sin\frac{2\pi k}{N})^2}} \angle -\tan^{-1}\frac{e^{-bT}\sin\frac{2\pi k}{N}}{1-e^{-bT}\cos\frac{2\pi k}{N}}$$

Homework Answers

- Problems 6.3b

$$\mathbf{X(k)} = \frac{1-e^{-bTN}}{\sqrt{(1-e^{-bT}\cos\frac{2\pi k}{N})^2 + (e^{-bT}\sin\frac{2\pi k}{N})^2}} \angle -\tan^{-1}\frac{e^{-bT}\sin\frac{2\pi k}{N}}{1-e^{-bT}\cos\frac{2\pi k}{N}}$$

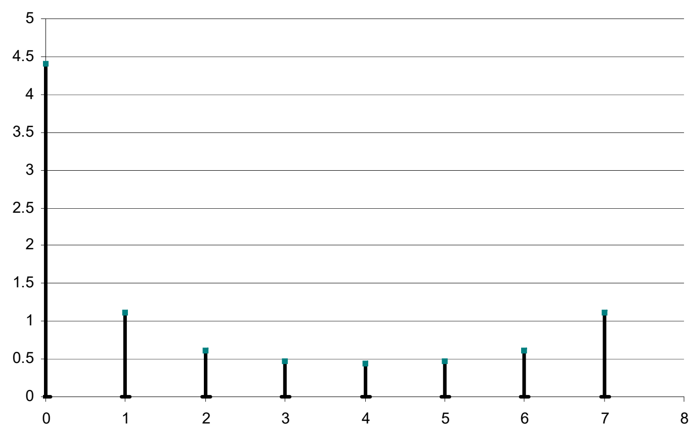




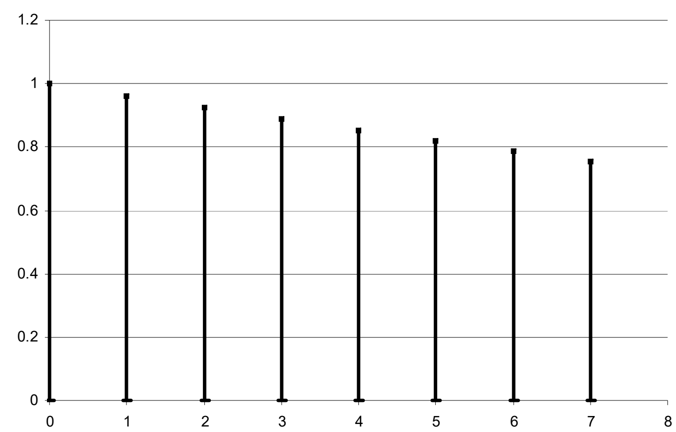
Homework Answers

- Problems 6.3c T=0.2,b=1, N=8, bT=.2

X(k)



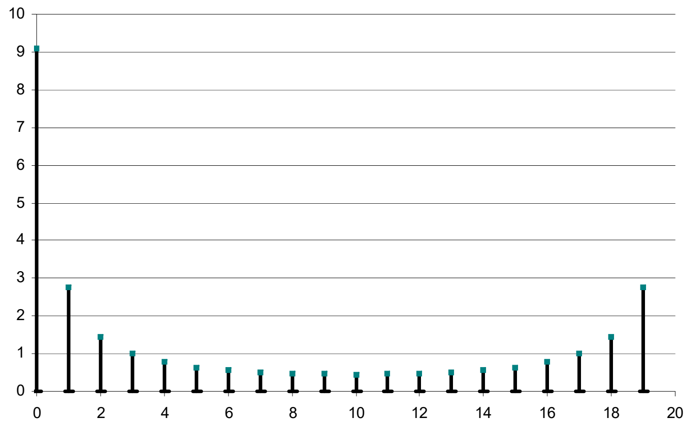
x(n)



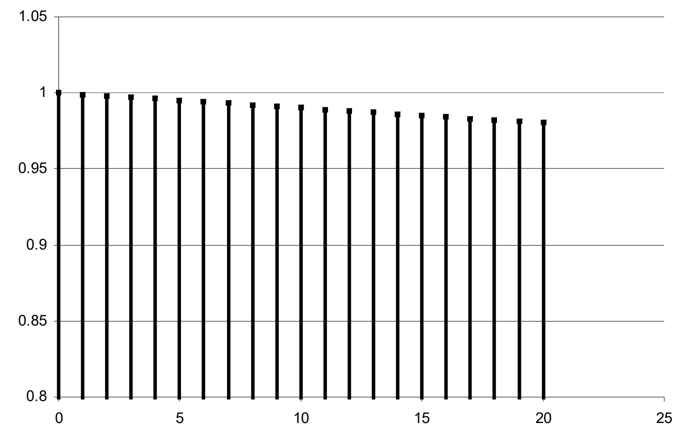
Homework Answers

- Problems 6.4 See 6.3 T=. 01,b=10, N=20, bT=1

X(k)

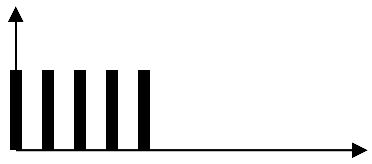


x(n)



Homework Answers

- Problems 6.6a



$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(n) = \sum_{n=o}^{N-1} \mathbf{X(k)} e^{jk2\pi n/N}$$

$$x(nT_s) = x[n] = 1; 0 \le n \le 4$$
$$= 0; n > 4$$

Note frequency spacing is $\frac{1}{5} \times f_s = \frac{1}{5} \times \frac{1}{T_s} = \frac{1}{5m} = 200 Hz$

$\mathrm{N} = 5$

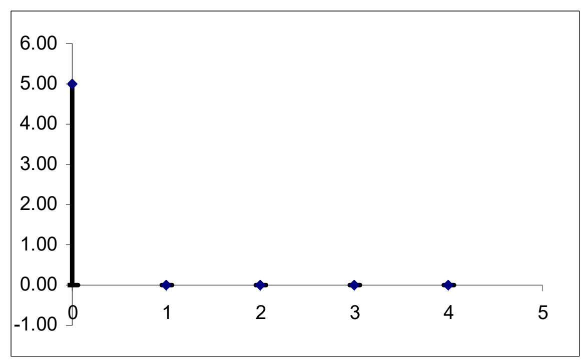
$$\mathbf{X(k)} = \sum_{n=0}^{4} e^{-jk2\pi n/5} = \sum_{n=0}^{4} (\cos k2\pi n/5 - j \sin k2\pi n/5)$$

$$= 1 + j0 + \cos k\frac{2\pi}{5} - j\sin k\frac{2\pi}{5} + \cos k\frac{4\pi}{5} - j\sin k\frac{4\pi}{5} + \cos k\frac{6\pi}{5} - j\sin k\frac{6\pi}{5} + \cos k\frac{8\pi}{5} - j\sin k\frac{8\pi}{5}$$

$$= 1 + \cos k\frac{2\pi}{5} + \cos k\frac{4\pi}{5} + \cos k\frac{6\pi}{5} + \cos k\frac{8\pi}{5} - j(\sin k\frac{2\pi}{5} + \sin k\frac{4\pi}{5} + \sin k\frac{6\pi}{5} + \sin k\frac{8\pi}{5})$$

N= 5

x(n)	1		1		1		1		1		X(k)
	n										
k	0		1		2		3		4		Mag
	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	
0	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	5.00
1	1.00	0.00	0.31	0.95	-0.81	0.59	-0.81	-0.59	0.31	-0.95	0.00
2	1.00	0.00	-0.81	0.59	0.31	-0.95	0.31	0.95	-0.81	-0.59	0.00
3	1.00	0.00	-0.81	-0.59	0.31	0.95	0.31	-0.95	-0.81	0.59	0.00
4	1.00	0.00	0.31	-0.95	-0.81	-0.59	-0.81	0.59	0.31	0.95	0.00



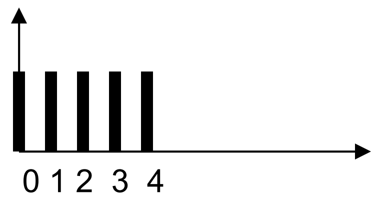
Homework Answers

- Problems 6.6a

$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(n) = \sum_{n=o}^{N-1} \mathbf{X(k)} e^{jk2\pi n/N}$$

$$x(nT_s) = x[n] = 1; 0 \le n \le 4$$
$$= 0; n > 4$$



$\mathrm{N} = 5$

OR

$$\mathbf{X(k)} = \sum_{n=0}^{4} e^{-jk2\pi n/5} = \sum_{n=0}^{4} \left(e^{-jk2\pi/5}\right)^n = \frac{1-\left(e^{-jk2\pi/5}\right)^5}{1-\left(e^{-jk2\pi/5}\right)} = \frac{1-e^{-jk2\pi}}{1-\left(e^{-jk2\pi/5}\right)} = 0; k \neq 0$$

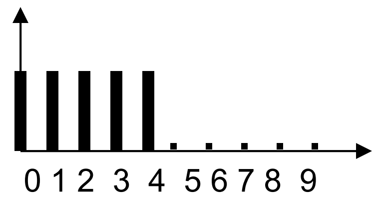
Using L'hopital's rule for $k = 0$

$$\underset{k=0}{Lim} \frac{1-e^{-jk2\pi}}{1-\left(e^{-jk2\pi/5}\right)} = \left.\frac{d\left(1-e^{-jk2\pi}\right)/dk}{d\left(1-\left(e^{-jk2\pi/5}\right)\right)/dk}\right|_{k=0} = \left.\frac{-(-jk2\pi)e^{-jk2\pi}}{-(-jk2\pi/5)e^{-jk2\pi/5}}\right|_{k=0} = 5$$

$$\mathbf{X(k)} = 5, \quad \text{for } k = 0$$
$$= 0, \quad \text{for } k \neq 0$$

Homework Answers

- Problems 6.6b



$$x(nT_s) = x[n] = 1; 0 \le n \le 4$$
$$= 0; n > 4$$

$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(n) = \sum_{n=o}^{N-1} \mathbf{X(k)} e^{jk2\pi n/N}$$

Note frequency spacing is $\frac{1}{10} \times f_s = \frac{1}{10} \times \frac{1}{T_s} = \frac{1}{10m} = 100Hz$

$\mathrm{N} = 10$

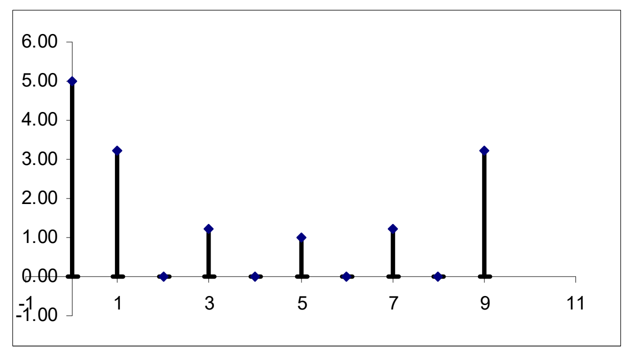
$$\mathbf{X(k)} = \sum_{n=0}^{9} x[n] e^{-jk2\pi n/10} = \sum_{n=0}^{4} e^{-jk2\pi n/10} = \sum_{n=0}^{4} (\cos k2\pi n/10 - j \sin k2\pi n/10)$$

$$= 1 + j0 + \cos k\frac{2\pi}{10} - j\sin k\frac{2\pi}{10} + \cos k\frac{4\pi}{10} - j\sin k\frac{4\pi}{10} + \cos k\frac{6\pi}{10} - j\sin k\frac{6\pi}{10} + \cos k\frac{8\pi}{10} - j\sin k\frac{8\pi}{10}$$

$$= 1 + \cos k\frac{2\pi}{10} + \cos k\frac{4\pi}{10} + \cos k\frac{6\pi}{10} + \cos k\frac{8\pi}{10} - j(\sin k\frac{2\pi}{10} + \sin k\frac{4\pi}{10} + \sin k\frac{6\pi}{10} + \sin k\frac{8\pi}{10})$$

N= 10

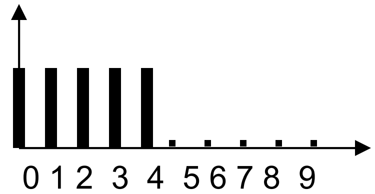
x(n)	1		1		1		1		1		0		0		0		0		0		X(k)
	n																				
k	0		1		2		3		4		5		6		7		8		9		Mag
	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	Real	Imag	
0	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00
1	1.00	0.00	0.81	0.59	0.31	0.95	-0.31	0.95	-0.81	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.24
2	1.00	0.00	0.31	0.95	-0.81	0.59	-0.81	-0.59	0.31	-0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	1.00	0.00	-0.31	0.95	-0.81	-0.59	0.81	-0.59	0.31	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.24
4	1.00	0.00	-0.81	0.59	0.31	-0.95	0.31	0.95	-0.81	-0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	1.00	0.00	-1.00	0.00	1.00	0.00	-1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
6	1.00	0.00	-0.81	-0.59	0.31	0.95	0.31	-0.95	-0.81	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	1.00	0.00	-0.31	-0.95	-0.81	0.59	0.81	0.59	0.31	-0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.24
8	1.00	0.00	0.31	-0.95	-0.81	-0.59	-0.81	0.59	0.31	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	1.00	0.00	0.81	-0.59	0.31	-0.95	-0.31	-0.95	-0.81	-0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.24
10	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00



Homework Answers

- Problems 6.6b

$$x(nT_s) = x[n] = 1; 0 \le n \le 4$$
$$= 0; n > 4$$
$$\mathrm{N} = 10$$
$$OR$$



$$\mathbf{X(k)} = \sum_{n=o}^{N-1} x(n) e^{-jk2\pi n/N}$$

$$x(n) = \sum_{n=o}^{N-1} \mathbf{X(k)} e^{jk2\pi n/N}$$

$$\mathbf{X(k)} = \sum_{n=0}^{9} e^{-jk2\pi n/10} = \sum_{n=0}^{4} e^{-jk2\pi n/10} = \sum_{n=0}^{4} \left(e^{-jk2\pi/10}\right)^n = \frac{1-\left(e^{-jk2\pi/10}\right)^5}{1-\left(e^{-jk2\pi/10}\right)}$$

$$\text{Note}: e^{-jk\pi} = (-1)^k$$

$$= \frac{1-e^{-jk\pi}}{1-\left(e^{-jk2\pi/10}\right)} = 0; k \ne 0, k \text{ even}$$

$$= \frac{2}{1-\left(e^{-jk2\pi/10}\right)} = \frac{2}{e^{-jk2\pi/5}(e^{jk2\pi/5} - e^{-jk2\pi/5})} = \frac{e^{jk2\pi/5}}{j} \frac{1}{\left(\frac{e^{jk2\pi/5} - e^{-jk2\pi/5}}{2j}\right)} = e^{j(k2\pi/5-\pi/2)} \frac{1}{\sin(k2\pi/5)}; k \text{ odd}$$

Using L'hopital's rule for $k = 0$

$$\lim_{k=0} \frac{1-e^{-jk\pi}}{1-\left(e^{-jk2\pi/10}\right)} = \left.\frac{d\left(1-e^{-jk\pi}\right)/dk}{d\left(1-\left(e^{-jk2\pi/10}\right)\right)/dk}\right|_{k=0} = \left.\frac{-(-jk\pi)e^{-jk\pi}}{-(-jk2\pi/10)\,e^{-jk2\pi/5}}\right|_{k=0} = 5$$

$$\mathbf{X(k)} = 5, \quad \text{for } k = 0$$
$$= e^{j(k2\pi/5-\pi/2)} \frac{1}{\sin(k2\pi/5)}, \ k \text{ odd}$$
$$= 0, \ \text{for } k \ne 0 \ \& \ \text{even}$$

Homework Answers

- Problems 6.2-4, 6

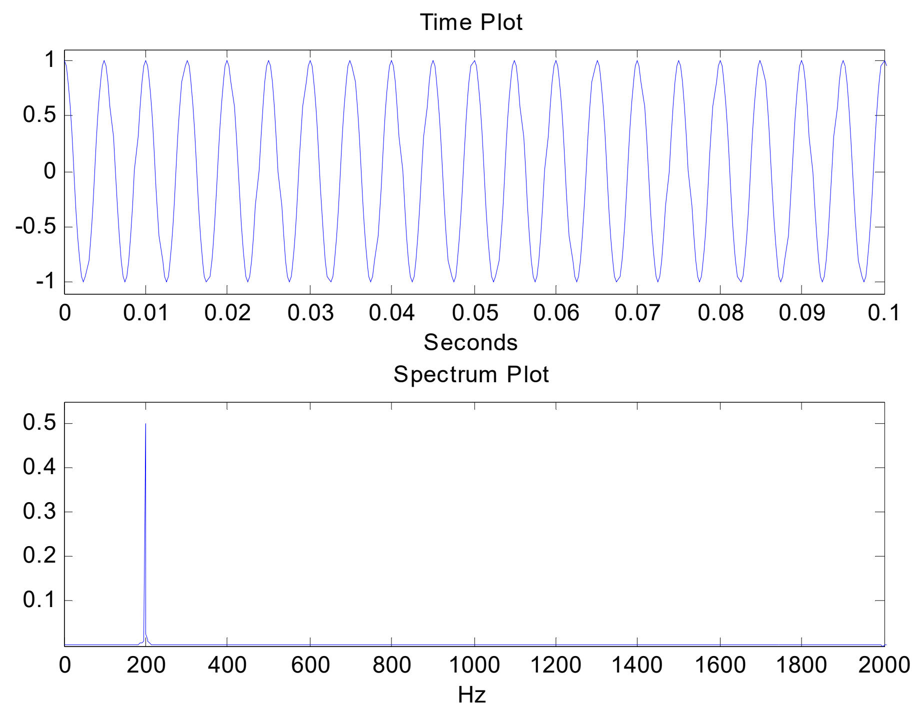
$$\sum_{0}^{N-1} a^n = 1 + a + a^2 + \cdots a^{N-1}$$

$$\frac{1}{1-a} = 1 + a + a^2 + \cdots a^{N-1} + \frac{a^N}{1-a}$$

$$\frac{1-a^N}{1-a} = 1 + a + a^2 + \cdots a^{N-1}$$

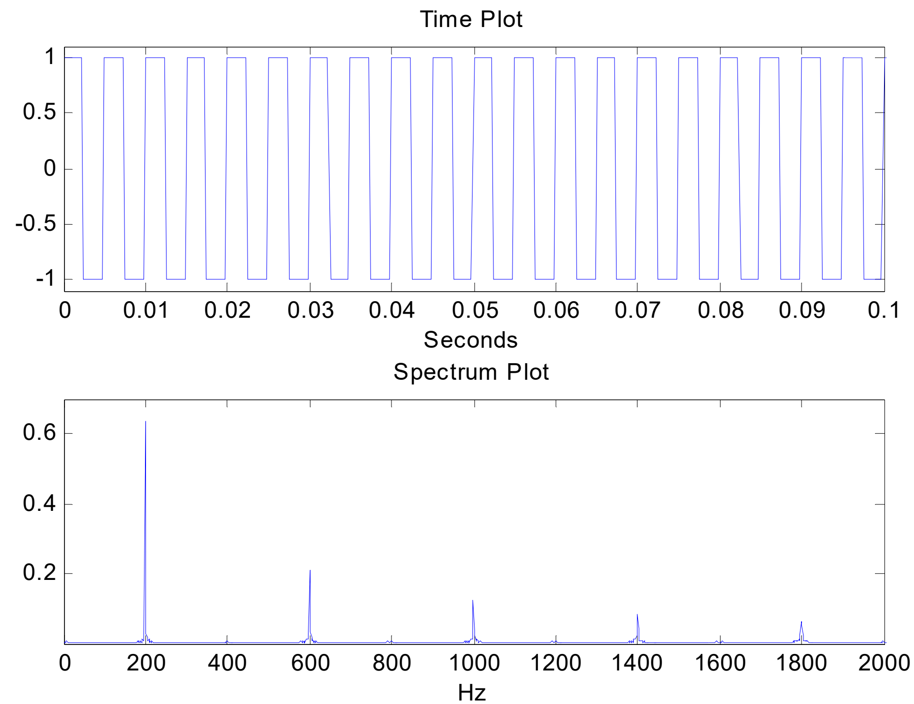
Matlab Code Single Sine Wave

```
fc=200;
to=1/fc;
fs=20*fc;
ts=1/fs;
cycles=100;
time=(0:ts:cycles*to);
N=length(time);
fo=fs/N;
x=cos(2*pi*fc*time);
subplot(2,1,1);
plot(time,x);
title('Time Plot');
xlabel('Seconds');
axis([0 cycles*to/5 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));
title('Spectrum Plot');
xlabel('Hz');
axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
```



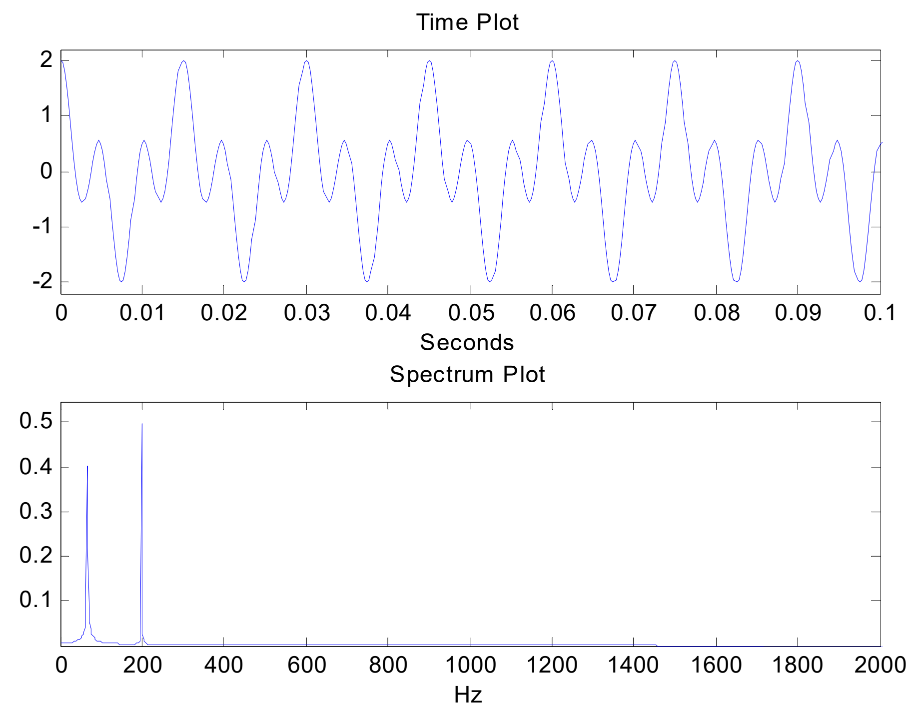
Matlab Code Single Square Wave

```
fc=200;
to=1/fc;
fs=20*fc;
ts=1/fs;
cycles=100;
time=(0:ts:cycles*to);
N=length(time);
fo=fs/N;
x=square(2*pi*fc*time);
subplot(2,1,1);
plot(time,x);
title('Time Plot');
xlabel('Seconds');
axis([0 cycles*to/5 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));
title('Spectrum Plot');
xlabel('Hz');
axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
```



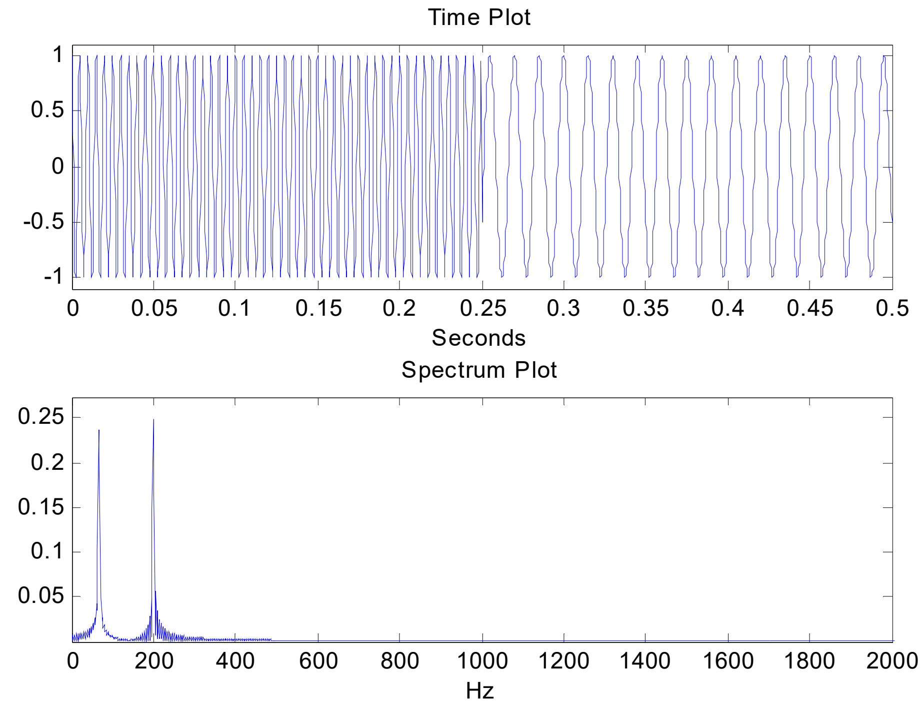
Matlab Code Simultaneous Sine Wave

```
fc=200;
to=1/fc;
fs=20*fc;
ts=1/fs;
cycles=100;
time=(0:ts:cycles*to);
N=length(time);
fo=fs/N;
x=cos(2*pi*fc*time)+cos(2*pi*fc/3*time);
subplot(2,1,1);
plot(time,x);
title('Time Plot');
xlabel('Seconds');
axis([0 cycles*to/5 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));
title('Spectrum Plot');
xlabel('Hz');
axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
```



Matlab Code Sequential Sine Wave

```
fc=200;
to=1/fc;
fs=20*fc;
ts=1/fs;
cycles=100;
time=(0:ts:cycles*to);
N=length(time);
fo=fs/N;
for i=1:N
   if i<N/2
      x(i)=cos(2*pi*fc*time(i));
   else
      x(i)=cos(2*pi*fc/3*time(i));
   end
end
subplot(2,1,1);
plot(time,x);
title('Time Plot');
xlabel('Seconds');
axis([0 cycles*to 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));
title('Spectrum Plot');
xlabel('Hz');
axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
```



Comparison

- Same basic spectrum and so you can't distinguish between the two. The amplitude of the sequential is half the size of the simultaneous because the time of occurrence of the each of the two signals is half in the sequential case compared to the simultaneous case